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Characterization of strength variability for reliability-based design of lime-cement columns

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Preface

The work presented in this thesis was conducted between January 2009 and March 2012 at the Division of Soil and Rock Mechanics, Department of Civil and Architectural Engineering, Royal Institute of Technology (KTH), Stockholm, Sweden. The work was supervised by Professor Stefan Larsson, Head of the Division of Soil and Rock Mechanics.

Sincere thanks are directed to the Development Fund of the Swedish Construction Industry (SBUF) and to the Swedish Transport Administration (Trafikverket), whose financial support made this work possible.

I would also like to thank my supervisor, Stefan Larsson, who has supported this work with his knowledge and never-ending enthusiasm. Further acknowledgments are directed to my colleagues at the Division of Soil and Rock Mechanics.

Finally, I would like to thank my family – my wife, Helena, for her support and understanding, my daughter, Greta, for the joy she brings me and my parents, Sture and Ann-Charlotte, for their assistance. Without you, this work would never have been possible.

Stockholm, March 2012

Niclas Bergman

Summary

An expanding population and increased need for infrastructure increasingly necessitate construction on surfaces with poor soil conditions. To facilitate the construction of buildings, roads and railroads in areas with poor soil conditions, these areas are often improved by means of foundation engineering. Constructions that are fairly limited in scope are often founded on shallow or deep foundations. However, these methods are relatively expensive and thus not applicable for large-scale constructions like roads and railroads. A cost-effective way to deal with poor soil conditions is to use ground improvement. This thesis deals with a ground improvement method called deep mixing (DM) using lime-cement columns.

Lime-cement columns are manufactured by pushing a mechanical mixing tool to the desired depth, with the tool then rotated and retracted while a lime-cement binder is distributed into soil, forming lime-cement columns. Because of the complex mixing process and inherent soil variability, soil improved by DM shows high variability with respect to strength and deformation properties. Due to this high variability, it is difficult to predict the properties in advance; hence it is important to verify the properties after installation. In Sweden, this is normally done using the column penetration test (KPS) method.

Current design praxis considers evaluated mean values in the design, and the effect of variability and uncertainties is dealt with by using a sufficiently high total factor of safety. A more rational approach for dealing with the effect of variability and uncertainties on the reliability of a mechanical system is to include them as parameters in the design model. This can be done by using reliability-based design (RBD). A major incentive for using RBD is that lower variability in design properties produces higher design values. This is important since it encourages contractors to improve their manufacturing methodologies because RBD allows more homogenous columns to be assigned higher design values. Reliability-based design is also in line with Eurocode 7, which states that the selection of the characteristic values for geotechnical parameters shall take the variability of the measured property values into account.

This licentiate thesis deals primarily with test methods and quantification of the strength variability of soil improved by lime-cement columns. The concept of RBD in serviceability limit state (SLS) design is briefly presented and discussed. Tip resistances from three different test sites using three different penetration test methods – the cone penetration test, the column penetration test and the total-sounding test – are analyzed and quantified in terms of means, variances and scale of fluctuations.

To summarize the most important findings and conclusions from this study:

- The scale of fluctuation was estimated to 0.2-0.7 m and 0-3 m in the vertical and horizontal direction, respectively.
- The relation between cone tip resistances measured using the cone penetration test and column penetration test does not correspond to the cone factors proposed in previous studies and in the Swedish Design Guidelines.
- The agreement between the column penetration test and total-sounding test was found to be “good enough”. Thus, it is suggested that the total-sounding test be used as a

complement to the column penetration test in evaluating the average strength properties of a group of medium- and high-strength lime-cement columns.

Sammanfattning

Med en växande population och infrastruktur, ökar behovet av att bebygga områden med dåliga grundläggningsförhållanden. För att möjliggöra byggandet av byggnader, vägar och järnvägar på dessa områden används olika typer av grundläggningsmetoder. Byggnationer med relativ liten utbredning kan ofta grundläggas med platta på mark eller pålning. Dessa grundläggningsmetoder är dock relativt dyra och därmed inte lämpliga för utbredda konstruktioner som vägar och järnvägar. Ett kostnadseffektivt alternativ till att handskas med dåliga grundläggningsförhållanden är olika jordförstärkningsmetoder. Denna avhandling behandlar jordförstärkningsmetoden djupstabilisering med kalk-cementpelare.

Kalk-cementpelare tillverkas genom att ett roterande blandningsverktyg trycks ner i jorden och ett bindemedel bestående av kalk och cement matas ut under omrörning. Variationerna i hållfasthets- och deformationsegenskaperna blir ofta stora på grund av den komplexa blandningsmekanismen samt variationerna i den naturliga jorden. På grund av de stora variationerna i hållfasthets- och deformationsegenskaperna är det svårt uppskatta dessa egenskaper innan tillverkning. Det blir således viktigt att man i efterhand kontrollerar sina antaganden avseende dessa egenskaper. I Sverige är den huvudsakliga provningsmetoden kalkpelarsondering.

Gällande dimensioneringsmetodik använder utvärderade medelvärden vid dimensionering där inverkan av variationer och osäkerheter hanteras med en tillräckligt stor säkerhetsfaktor. Ett mer rationellt tillvägagångssätt att ta hänsyn till inverkan av variationer och osäkerheter på säkerheten i en konstruktion, är att ta med dem som parametrar i designmodellen. Detta görs möjligt genom sannolikhetsbaserad dimensionering. Ett av de främsta incitamenten till införandet av sannolikhetsbaserad dimensionering är att lägre variationer i en egenskap leder till ett högre dimensionerande värde. Detta är väsentligt eftersom det uppmuntrar entreprenörer till att utveckla sina produktionsmetoder då sannolikhetsbaserad dimensionering tillåter att mer homogena pelare ges högre dimensionerande värde. Ett ytterligare incitament till införandet av sannolikhetsbaserad dimensionering är att den uppfyller kraven i Eurocode 7 som gör gällande att man vid utvärderingen av karakteristiska värden ska ta hänsyn till variationerna hos den uppmätta parametern.

Denna licentiatuppsats behandlar främst test metoder och kvantifiering av variationer i hållfasthetsparametrar i jord förstärkt med kalk-cement pelare. Begreppet sannolikhetsbaserad dimensionering i brukgränstillståndet presenteras och diskuteras övergripande. Spetstrycket från tre olika testmetoder, kalkpelarsonden, CPT-sonden samt Jb-totalsonden, utförda i kalk-cementpelare på tre olika testplatser, kvantifieras avseende medelvärden, varianser samt fluktuationsavstånd.

De viktigaste upptäckterna och slutsatserna från denna studie kan summeras enligt:

- Fluktuationsavståndet uppmättes till 0.2-0.7 m i vertikalled och 0-3 m i horisontalled.
- Förhållandet mellan uppmätta spetstryck från CPT-sonden och kalkpelaresonden överensstämmer inte med de bärighetsfaktorer som föreslagits i tidigare studier och i svensk standard.
- Överensstämmelsen mellan kalkpelaresonden och Jb-totalsonden var tillräcklig för att Jb-totalsonden ska kunna användas som ett komplement till kalkpelarsonden för att uppskatta medelhållfastheten i en grupp med hårda och medelhårda kalk-cementpelare.

List of publications

This licentiate thesis is based on work presented in the following publications.

Appended papers:

Paper I Al-Naqshabandy, M. S., Bergman, N. and Larsson, S., 2012. Strength variability in lime-cement columns based on CPT data. *Ground Improvement* 165 (1), 15-30.

Al-Naqshabandy and Bergman performed the analyses in parallel. Al-Naqshabandy, Bergman and Larsson jointly wrote the paper.

Paper II Bergman, N., Al-Naqshabandy, M. S. and Larsson, S. Strength variability in lime-cement columns evaluated using CPT and KPS. *Submitted to* Georisk *in November 2011.*

Bergman performed the analyses and wrote the paper. Al-Naqshabandy contributed valuable comments. Larsson contributed writing and valuable comments.

Paper III Bergman, N. and Larsson, S. Agreement between KPS and Jbt data evaluated from soil improved by lime-cement columns. *Submitted to* Ground Improvement *in April 2012.*

Bergman performed the analyses and wrote the paper. Larsson contributed writing and valuable comments.

Connecting reports:

Fransson, J., 2011. A study of the correlation between soil-rock sounding and column penetration test data. KTH, Master Thesis.

Bergman supervised the work.

Ehnbom, V. and Kumlin, F., 2011. Reliability-Based Design of Lime-Cement Columns based on Total Settlement Criterion. KTH, Master Thesis.

Bergman supervised the work.

Contents

Chapter 1 – Introduction

1.1 Background

An expanding population and growing need for infrastructure increasingly necessitate construction on surfaces with poor soil conditions. To facilitate the construction of buildings, roads and railroads in areas with poor soil conditions, these areas are often improved by means of ground improvement, shallow or deep foundations. Constructions that are fairly limited in scope are often founded on shallow or deep foundations. However, these methods are relatively expensive and thus not applicable to large-scale constructions like roads and railroads. A cost-effective way to deal with poor soil conditions is to use ground improvement methods. This thesis deals with a ground improvement method called deep mixing (DM) by lime-cement columns.

Deep mixing by lime-cement columns is a ground improvement method developed simultaneously in the Scandinavian countries and Japan during the 1970's (Boman and Broms 1975; Broms 1984; Terashi and Juran 2000; Larsson 2005a). The method is mainly applicable to soft soils like clay, silt and peat and improves the strength and deformation properties of the soil. Columns are manufactured by pushing a mechanical mixing tool to the desired depth. The mixing tool is then rotated and retracted while a binder is distributed into the soil, forming columns. Deep mixing can be subdivided into two groups, depending on how the binder is distributed (Topolnicki 2004; Larsson 2005a). The method commonly used in Sweden is known as the dry method and uses compressed air to distribute the dry binder powder into the soil. Another category of DM methods is the wet mixing method, where the binder, normally cement, is mixed with water prior to installation. In this thesis, the dry mixing method is studied. Figure 1(a) shows typical machinery for manufacturing lime-cement columns, and Figure 1(b) shows two mechanical mixing tools.

Because of the complex mixing process and inherent soil variability, soil improved by DM shows high variability with respect to strength and deformation properties (Larsson 2005a). Due to the high variability, it is difficult to predict the properties in advance; hence it is important to verify the properties after installation. In Sweden, this is normally done using the column penetration test method (KPS) (Axelsson and Larsson 2003; TK Geo 2011).

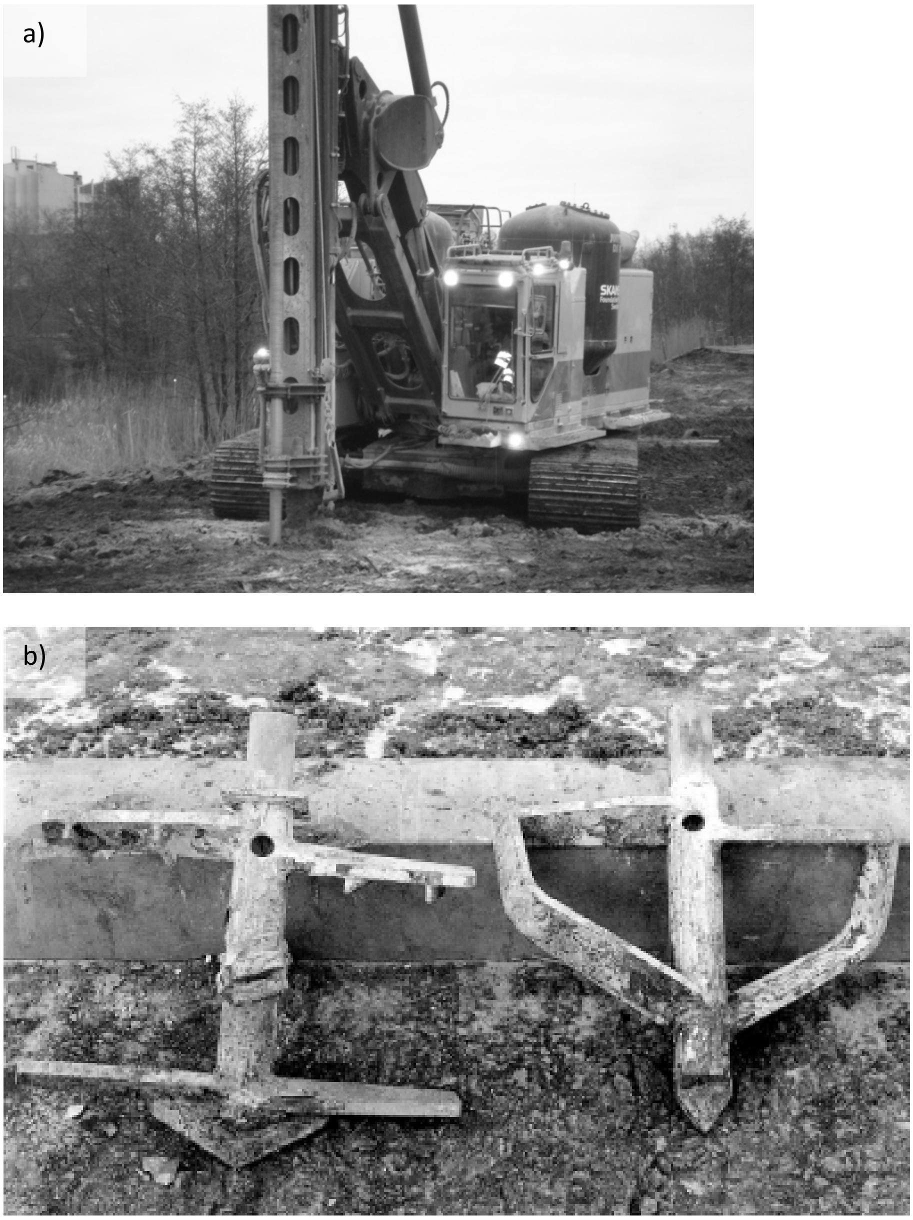

Figure 1: (a) Typical machinery for manufacturing lime-cement columns (b) Pin drill and Swedish standard mixing tool (courtesy of Skanska AB; Larsson et al. 2005).

Current design praxis uses deterministic mean values for design, and uncertainties in evaluating the mean are incorporated in a single value represented by a partial factor or total factor of safety. This means that a high-quality column with low-strength variability is assigned a design value equal to that of a low-quality column with high-strength variability, provided that the average strength of the columns is equal. One problem with this design approach is that it does not promote improvement in column quality since there is nothing to be gained from this, at least from a manufacturer's point of view. A more rational design approach would be to incorporate the uncertainties as parameters in the design model. This can be done by introducing reliability-based design (RBD). The reliability-based design approach promotes the development of manufacturing methodologies since it assigns relatively higher design values to high-quality columns. Furthermore, RBD is in line with Eurocode 7 (Eurocode 7: Geotechnical design – Part 1: General rules 2004), which states that the selection of characteristic values of geotechnical parameters shall take the variability of the property values measured into account.

1.2 Previous studies

Although RBD is rarely used in practice, the need for it in DM has been identified by several authors. This section gives a brief overview of previous studies published in journals, at conferences or in theses, addressing the RBD of soil improved by DM. Previous studies examining the strength variability of soil improved by DMdue to the significant impact of variability on RBD are presented RBD.

Honjo (1982) was the first to address the need for RBD in DM. Honjo proposed a probabilistic failure model taking the variability of the soil into account in design. Statistical methods were used to quantify the compressive strength and variability of soil improved by DM. The scale of fluctuation was evaluated by means of an autocorrelation function. It was concluded that the coefficient of variation of the unconfined compressive strength of the stabilized soil ranges between 0.21 and 0.36, regardless of the average strength. Furthermore, the scale of fluctuation in a vertical direction is influenced by factors such as in-situ soil properties, binder content and mixing conditions.

Filz and Navin (2006) presented the concept of RBD in the ultimate limit state (ULS) design of column-supported embankments. Reliability-based design is recommended for a DM project, mainly since it accounts for the significant variability in deep-mixed materials, but also since it permits rational development of statistically based design specifications. Furthermore, the study presents a coefficient of variation of unconfined compressive strength from nine deep-mixing projects in the U.S. ranging between 0.34 and 0.79.

Further studies addressing RBD in ULS design of soil improved by DM are presented by Kitazume (2004), Navin (2005), Terashi and Kitazume (2009), Kasama et al. (2009) and Adams et al. (2009).

Babu et al. (2011) illustrates the use of a reliability analysis for unconfined compressive strength of soil improved by DM. They conclude that reliability-based analyses provide a rational choice of design strength values.

Navin and Filz (2005) analyzed thirteen data sets of unconfined compressive strength for deep-mixed materials constructed using the wet and dry method. Their analysis showed that strength data tend to fit a log-normal distribution. Values of the coefficient of variation of the unconfined compressive strength ranged from 0.34 to 0.74. Analyses of the spatial correlation indicated a scale of fluctuation of 12 m for the wet method. For the dry method, no scale of fluctuation could be detected. Furthermore, a moderate correlation was found between the unconfined compressive strength and the elastic modulus.

Larsson et al. (2005a) and (2005b) investigated the influence of a number of factors in the installation process on the strength variability of lime-cement columns. The retrieval rate and the number of mixing blades were found to have a significant impact on variability, while rotational speed, binder tank air pressure and diameter of the outlet hole were insignificant.

The variability of soil improved by DM has further been studied by Hedman and Kuokkanen (2003), Larsson et al. (2005c) and Larsson and Nilsson (2009), among others.

While a number of papers about RBD in DM have been published, RBD DMno study addresses RBD in SLS design. Furthermore, a number of papers address inherent and spatial variability in DM, although other sources of uncertainties (such as measurement, statistical and model transformation uncertainties) have not been considered.

1.3 Scope of research

The main objective of this research project is to implement a reliability-based design methodology in the serviceability limit state design of soil improved by lime-cement columns. The first part of this project, which is presented in this licentiate thesis, deals primarily with test methods and the quantification of strength variability. The actual implementation of RBD in SLS design will be conducted during the second part of this project, and the concept of RBD in SLS is presented and discussed only briefly in this thesis.

The scope of this thesis can be summarized as follows:

- Make a contribution to the empirical knowledge about strength variability in soil improved by DM and its influence in determining the design value using RBD.
- Investigate the influence of different test methods (the cone penetration test and column penetration test) on the quantification of means, variances and scale of fluctuations.
- Investigate the possibility of using the total-sounding test method to assess the strength of soil improved by DM.
- Investigate how current deterministic SLS design methodology can be used in RBD.

1.4 Outline of thesis

This thesis consists of an introductory section in which the background and objectives of this study are presented. A summary of the literature survey is presented, including major findings and conclusions from previous work.

Chapter 2 – Quality control

This chapter gives an introduction to current Swedish quality control methodology. It also presents the penetration test methods used in this study.

Chapter 3 – Statistical analyses

Using RBD, a statistical quantification of the mean value and uncertainties related to the evaluation of the mean value is essential. This chapter presents the statistical analyses used in this study. Furthermore, the concept of variance reduction is introduced, and correlation and agreement analyses are explained.

Chapter 4 – Uncertainties

Using RBD, the impact of uncertainties on the determination of the design value is significant. This chapter gives an introduction to uncertainties in general and to uncertainties related to DM in particular.

Chapter 5 – Reliability-based design

This chapter gives an introduction to the concept of RBD. Furthermore, an example is given of how RBD can be incorporated in the serviceability limit state design of soil improved by DM.

Chapter 6 – Summary of appended papers

This chapter gives a brief summary of the appended papers.

Chapter 7 – Conclusions

This chapter summarizes the major conclusions from this study and gives suggestions for future work related to this study.

Chapter 2 – Quality control

In this study, the quality of lime-cement columns was studied using three different penetration test methods. This chapter gives an introduction to current Swedish quality control methodology. Furthermore, the three different penetration test methods used in this study are presented.

2.1 General

Because of the complex mixing process, the variability of the column strength properties is normally very high, which is why it is difficult to predict the quality of the columns in advance (Larsson 2005a). The quality of lime-cement columns is governed by several factors, such as the rheology of the soil and binder, stress conditions in the soil, geometry of the mixing tool and retrieval rate of the mixing tool. Although the influence of these factors on the quality of the lime-cement column has been investigated by Larsson et al. (2005a, 2005b), it is not considered in practice. Consequently, it is important to test the quality of the columns after installation. In Sweden 1% or at least four of the columns are tested after installation (TK Geo 2011; AMA Anläggning 10).

2.2 Test methods

In Sweden, the most frequently used penetration test method is the column penetration test (KPS). Internationally, a wide range of field test methods have been used, such as the reversed column penetration test ($OKPS$), cone penetration test (CPT), standard penetration test (SPT), rotary sounding test (RPT) and pressure meter test (PMT) (Porbaha 2002). In this thesis, data from three different test methods – the column penetration test, cone penetration test and total-sounding test (Jbt) – were analyzed.

2.2.1 Column penetration test (KPS)

The column penetration test was developed in the 1980's by Torstensson (1980a, 1980b) and is the most frequently used penetration test method in Sweden today for quality control of lime-cement column properties. The test is executed by pushing a cylindrical penetrometer with two horizontal vanes, or probe (see Figure 2), down into the center of the column, while continuously recording the penetration force (Q_{KPS}). Tests are normally performed according to Swedish guidelines (TK Geo 2011, Larsson 2006). The probe is pushed into the column at a constant rate of penetration of 20 mm/sec. To get a good representation of the column, the probe should be as wide as possible and preferably 100 mm smaller than the column diameter (Axelsson and Larsson 2003). Because of the relatively large size of the KPS-probe, it is recommended for depths of no more than 8 m (Larsson 2006). At greater depths and in high-strength columns, the probe easily deviates from the column. To facilitate the verticality of the KPS-probe, a center hole can be bored in the column. In so doing, the penetration depth of the KPS-probe may be increased to 12-15 m (Ekström 1994). The column penetration test can be improved by attaching the KPS-probe to a cone penetration test (CPT). This improvement is important since it enables KPS to distinguish bar friction from penetration resistance ($q_{c,KPS}$), where bar friction can be as large as $q_{c,KPS}$ in stabilized soil (Larsson 2005a).

From $q_{c,KPS}$ the column undrained shear strength (c_u) can be evaluated using the following empirical relation:

$$c_u = \frac{q_{c,KPS}}{N_{k,KPS}} \quad (1)$$

where $N_{k,KPS}$ is the cone factor for KPS. According to Swedish guidelines $N_{k,KPS}$ should be set at 10. However, values for $N_{k,KPS}$ ranging from 10 to 20 have been suggested by several authors (Liyanapathirana and Kelly 2011; Wiggers and Perzon 2005; Axelsson 2001; Halkola 1999).

2.2.2 Cone penetration test (CPT)

The cone penetration test is a penetration test method used internationally to test improved soil (Halkola 1999; Larsson 2005a, 2005b; Puppala et al. 2005a, 2005b). The cone penetration test used in this study included a cylindrical electronic test probe whose cone tip measured 1000 mm^2. Like the KPS, the CPT probe (Figure 3) is driven into the column using a constant rate of penetration of 20 mm/sec. The penetration resistance ($q_{c,CPT}$) is continuously measured, and c_u can be evaluated using the following empirical relation (Lunne et al. 1997):

$$c_u = \frac{q_{c,CPT} - \sigma_{v0}}{N_{k,CPT}} \quad (2)$$

where $N_{k,CPT}$ is the cone factor for CPT and σ_{v0} is the total vertical soil stress. Values for $N_{k,CPT}$ ranging from 15 to 23 have been suggested by several authors (Porbaha 2001; Puppala et al. 2005b; Tanaka et al. 2000).

Figure 2: The column penetration test (KPS) (courtesy of Geotech).

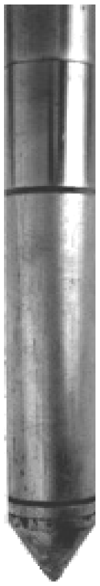

Figure 3: The cone penetration test (CPT).

2.2.3 Total-sounding test (Jbt)

The Swedish total-sounding test method is a modification of the Norwegian total-sounding test method (SGF 2006). It was primarily designed to measure bedrock level and to determine the existence of large boulders and has been used successfully to locate and map the extent of quick clay formations (Lundström et al. 2009; Solberg et al. 2011). Furthermore, Jbt has been used to evaluate lime-cement column strength properties (Jelisic and Nilsson 2005; Nilsson and Forssman 2004). Tsukada et al. (1998) used the rotary penetration test (Porbaha 2002), a test method similar to Jbt, to evaluate the strength of improved soil. The total-sounding test method is a rotary penetration test where a vertical force is applied to a rotating drilling rod. Standard equipment is a 57 mm drill bit (Figure 4) attached to a 44 mm drilling rod. The rod is driven into the center of the lime-cement column with a rate of penetration of 20 mm/s and with a rotational speed of 25 rpm, while continuously recording the penetration force (Q_{Jbt}). In addition to the tip penetration resistance ($q_{c,Jbt}$), drill rod bar friction is also included in Q_{Jbt}. This is important to consider since drill rod bar friction may constitute a large part of Q_{Jbt} in improved soil.

Figure 4: The total-sounding (Jbt) bore bit (courtesy of Geotech).

Chapter 3 – Statistical analyses

Using RBD, a statistical quantification of the mean value and uncertainties related to the evaluation of the mean value is essential. This chapter presents the statistical analyses used in this study. Furthermore, the concept of variance reduction is introduced, and correlation and agreement analyses are explained.

3.1Spatial variability

An important measure of soil variability is spatial variability. Spatial variability can be described as the variability of a mean value in space. In order to quantify spatial variability, three statistical measures are needed – the mean, the variance and the scale of fluctuation.

3.1.1 Mean

The arithmetic mean ($\bar{x}$) is a numerical measure to describe a set of data. It is defined as the sum of the observations divided by sample size. It is defined by the following formula:

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \tag{3}$$

where x_i is the i^{th} observation and n the number of observations.

3.1.2 Variance

The most common measure of the variation of a set of data is the sample variance (s^2). It measures the degree to which the actual values differ from the mean and is defined by the following formula:

$$s^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})}{n-1} \tag{4}$$

It can also be quantified as the coefficient of variation (COV), which is defined by the following formula:

$$COV = \frac{\sqrt{s^2}}{\bar{x}} \tag{5}$$

3.1.3 Scale of fluctuation

The scale of fluctuation (θ) is an important measure in evaluating spatial variability and can be described as the distance within which a measured parameter shows a relatively strong correlation (Vanmarcke 1977). The occurrence of θ has a significant impact on the evaluation of the mean. If a series of measurements lie closer than θ, we can expect that the average of the measurements is probably higher or lower than the average of the soil layer tested. The scale of fluctuation is commonly evaluated using variograms or autocorrelation functions (ACF). In the study, θ was evaluated from the sample ACF, which is the variation of the autocorrelation coefficient ($\rho'(k)$):

$$\rho'(k) = \frac{c_k}{c_0} \tag{6}$$

where c_k is the autocovariance at lag distance k and c_0 is the autocovariance at lag distance 0 . c_k is defined by:

$$c_k = Cov(q_c(z_i), q_c(z_{i+k})) = E[(q_c(z_i) - \overline{q_c})(q_c(z_{i+k}) - \overline{q_c})] \tag{7}$$

where k is the lag distance, $q_c(z_i)$ is the tip resistance at depth z_i, i = 0,1,… n-1 and $\overline{q_c}$ is the mean tip resistance.

By fitting a theoretical ACF ($\rho(k)$) into the sample ACF, the one-dimensional θ can be evaluated by (Vanmarcke 1983):

$$\theta = 2\int_0^\infty \rho(k)dk \tag{8}$$

Five theoretical models are widely used in analyzing geotechnical data as shown in the table below (Table 1) (Jaksa 1999; Phoon 2003). Due to best fit and the relatively limited data, the binary noise model was used in this study.

Table 1: $\rho(k)$ is the theoretical autocorrelation function, k is the lag distance, and c, m, b, d and a are model constants (decay factors).

Autocorrelation model	**Equation**
Binary noise	$\rho(k) = \begin{cases} 1 - c\lvert k\rvert & k \le 1/c \\ 0 & otherwise \end{cases}$
Single exponential	$\rho(k) = \exp\,(-m\lvert k\rvert)$
Squared exponential	$\rho(k) = \exp\,(-bk)^2$
Cosine exponential	$\rho(k) = \exp(-\mathrm{d}\lvert\mathrm{k}\rvert)\cos\,(\mathrm{d}\lvert\mathrm{k}\rvert)$
Second-order Markov	$\rho(k) = (1 + a\lvert k\rvert)\exp\,(-a\lvert k\rvert)$

3.2 Variance reduction factor

The effect of spatial variability on the determination of the design value can be dealt with through spatial averages, in this study represented by the average tip resistance over a depth or volume. The variance reduction factor (Γ^2) is dependent on θ and the scale of scrutiny (L), that is, the size of the mechanical system of failure domain, where a small θ and a large L are attributes that contribute to a reduction in variability. Vanmarcke (1977) defines Γ^2 in the one-dimensional case as:

$$\Gamma^2(L_x) = \frac{2}{L_x}\int_0^{L_x}\left(1 - \frac{k}{L_x}\right)\rho(k)dk \tag{9}$$

where L_x is the size of the average length of the domain size, k is the separation distance and $\rho(k)$ is the normalized autocorrelation function. Assuming separate correlation structures, the three-dimensional Γ^2 is defined as:

$$\Gamma_{xyz}^2 = \frac{2\cdot2\cdot2}{L_x\cdot L_y\cdot L_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\left[\left(1 - \frac{x}{L_x}\right)\left(1 - \frac{y}{L_y}\right)\left(1 - \frac{z}{L_z}\right)\rho(x)\rho(y)\rho(z)\right]dydxdz \tag{10}$$

The use of Γ^2 will be further described in section 4.2.

3.3 Correlation and agreement

In order to investigate the possibility of using Jbt to assess the strength of soil improved by DM, the correlation and agreement between Jbt and KPS are analyzed.

Correlation analysis is a widely used tool for quantifying the relation between two or more sets of data. A commonly used measure is the Pearson product-moment correlation coefficient. It gives a measurement of linear correlation and can be estimated by the sample correlation coefficient (r) according to:

$$r = \frac{1}{n-1} \sum_{i=1}^{n} \left(\frac{x_i - \bar{x}}{s_x}\right) \left(\frac{y_i - \bar{y}}{s_y}\right) \qquad (11)$$

where n is the number of data in a sample, x and y are two sets of data, $\bar{x}$ and $\bar{y}$ are the mean values, and s_x and s_y are the sample standard deviation of the respective sets. It can be shown that the value of $r$is always between -1 and 1. A value of $r = 1$ implies a perfect positive linear relation between x and y, while $r = -1$ implies a perfect negative linear relation. A value of $r = 0$ implies that there is no linear relation between x and y.

Correlation analysis not is always a good measure of agreement between two sets of data. There will be a perfect correlation if the data scatter plot follows any straight line, but there will be perfect agreement only if the data scatter plot follows the line of perfect equality (Figure 5).

The agreement between two sets of data can be visualized by Tukey mean-difference plots (Tukey 1977) (Figure 6), where the differences between data points are plotted against their average values. However, the extent to which the two measurements can differ without having a significant impact on the evaluation of column undrained shear strength will be a question of judgment. The Tukey mean-difference plot is only meaningful for two similar sets of test data, that is, with the same physical dimensions and expressed in the same units.

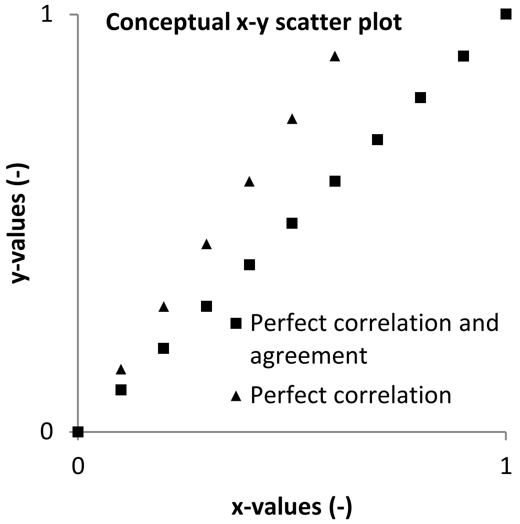

Figure 5: Perfect correlation is obtained if the data scatter plot follows any straight line; perfect agreement will be obtained only if the data scatter plot follows the line of perfect equality.

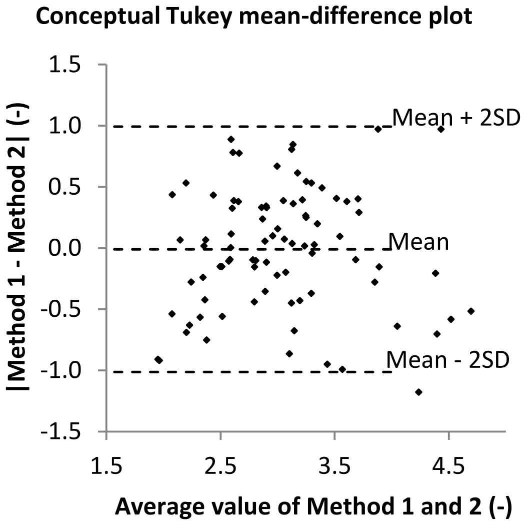

Figure 6: Conceptual Tukey mean-difference plot, where differences between data points are plotted against their average values.

Chapter 4 – Uncertainties and their impact on the evaluation of the design value

4.1 General

Geotechnical engineers face many sources of uncertainties in the design process (Phoon and Kulhawy 1999a, 1999b; Baecher and Christian 2003). Design parameters are often evaluated from field and laboratory tests using empirical relations. Figure 7 categorizes the different sources of geotechnical uncertainties. Geotechnical uncertainties can be described as either aleatory or epistemic. Aleatory uncertainties are those associated with randomness, or are modeled as caused by chance. In geotechnical engineering, data scatter from laboratory and field tests is often modeled as caused by chance. Furthermore, data scatter from tests is considered to be caused by natural variability in the soil and measurement errors. Epistemic uncertainties, commonly known as knowledge uncertainties, are associated with a lack of information or knowledge about processes and physical laws that limits our ability to model the real world. Transformation or model errors and statistical errors are examples of epistemic uncertainties. Transformation or model errors are often associated with the accuracy and validity of empirical relations, such as Equation 1 and 2. Statistical errors are associated with the precision with which model parameters can be estimated, and are governed by available test data. In this study, uncertainties are quantified by means of COV.

In Sweden, the effect of parameter uncertainties on the design of geotechnical constructions has been studied previously by Olsson (1986), Alén (1998) and Stille et al. (2003, among others.

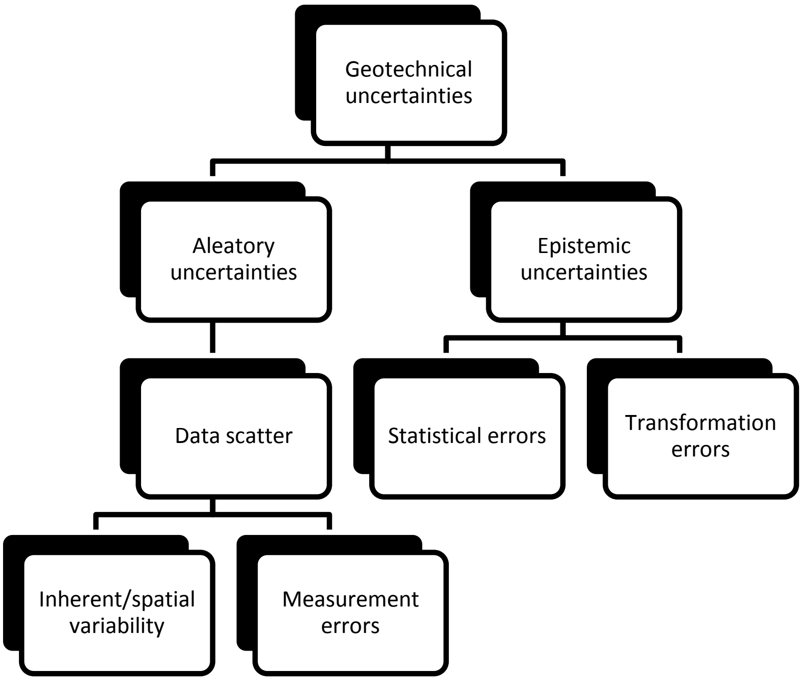

Figure 7: Classification of different sources of geotechnical uncertainties (after Baecher and Christian (2003)).

4.2 Uncertainties in deep mixing

For natural soils, soils improved by deep mixing have a relatively high inherent variability. The high variability is mainly caused by the complex mixing process and natural variability of the unimproved soil (Larsson 2005a). Larsson (2005a), Burke and Sehn (2005), Navin and Filz (2005) and Kasama and Zen (2009) present COV evaluated from compression tests of soil improved with DM ranging from 14 to 76%.

Another major source of uncertainties in DM is transformation errors. Deformation properties, such as undrained shear strength, are often evaluated from their empirical relation with the cone tip resistance of a penetration test method. In Equation 1 and 2, the relation is governed by a cone factor ($N_{k,KPS}$ and $N_{k,CPT}$). However, the wide range of cone factors suggested for the two methods introduces further uncertainties into the evaluation.

In evaluating the average column undrained shear strength ($\bar{c}_{u,col}$), uncertainties can be modeled as stochastic variables, representing quotas of the parameter measured:

$$\bar{c}_{u,col} \propto \bar{q}_c \cdot (\eta_w \cdot \eta_m \cdot \eta_{st} \cdot \eta_{tr}) \qquad (12)$$

where $\bar{q}_c$ is the average tip resistance, η_w is the uncertainty associated with spatial variability, η_m is the uncertainty associated with measurement errors, η_{st} is the uncertainty associated with statistical errors and η_{tr} is the uncertainty associated with transformation errors. The quotas are assumed to be normally distributed with an expected value and a standard deviation according to:

$$\eta_w \in N(1, COV_w) \qquad (13)$$

$$\eta_m \in N(1, COV_m) \qquad (14)$$

$$\eta_{st} \in N(1, COV_{st}) \qquad (15)$$

$$\eta_{tr} \in N(N_k, N_k \cdot COV_{tr}) \qquad (16)$$

where COV_w is the coefficient of variation associated with inherent variability, COV_m is the coefficient of variation associated with measurement errors, COV_{st} is the coefficient of variation associated with statistical errors and COV_{tr} is the coefficient of variation associated with transformation errors.

The uncertainty of a product of stochastic variables can be approximated by the square root of the sum of the squared COV of individual stochastic variables (Goodman 1960; Jaksa et al. 1997). Consequently, the total uncertainty ($COV_{\overline{c_u}}$) in determining the design value, evaluated from mean tip resistances ($\overline{q_c}$), can be defined as:

$$COV_{\overline{c_u}} = \sqrt{COV^2_{w,\overline{q_c}} + COV^2_{m,\overline{q_c}} + COV^2_{st,\overline{q_c}} + COV^2_{tr,\overline{q_c}}} \qquad (17)$$

Based on penetration test data, the COV of individual sources of uncertainties is given by:

$$COV^2_{w,\overline{q_c}} = \left(COV^2_{qc} - COV^2_{m,q_c}\right) \cdot \Gamma^2 \qquad (18)$$

$$COV^2_{m,\overline{q_c}} = \frac{COV^2_{m,q_c}}{N} \qquad (19)$$

$$COV^2_{st,\overline{q_c}} = \left(COV^2_{qc} - COV^2_{m,q_c}\right) \cdot \frac{1}{N} \tag{20}$$

where COV_{qc} is the evaluated coefficient of variation of tip resistance (q_c), COV_{m,q_c} is the coefficient of variation associated with random measurement noise and N is the number of uncorrelated tests with respect to $\overline{q_c}$.

Combining Equation 17–20, Equation 17 can be re-written as:

$$COV^2_{\overline{c_u}} = \left(COV^2_{qc} - COV^2_{m,q_c}\right)\left(\frac{1}{N} + \Gamma^2\right) + \frac{COV^2_{m,qc}}{N} + COV^2_{tr,\overline{q_c}} \tag{21}$$

4.3 Evaluation of design value

Using a RBD methodology, uncertainties are included as design parameters in the evaluation of the design value. For normally distributed variables, the design value can be calculated as (Thoft-Christensen and Baker 1982):

$$x_d = \bar{x} + \alpha\beta\sqrt{s^2} \tag{22}$$

where $\bar{x}$ is the sample mean, s^2 is the sample variance, α is the sensitivity factor that describes the significance of the variable for the mechanical system and β is the required reliability index.

Based on penetration test data, the normalized design value ($c_{u,d}/\overline{c_u}$) can be evaluated as:

$$c_{u,d}/\overline{c_u} = 1 + \alpha\beta \cdot \sqrt{COV^2_{\overline{c_u}}} \tag{23}$$

For log-normally distributed variables, Equation 23 can be re-written as:

$$c_{u,d}/\overline{c_u} = exp\left[-\frac{1}{2}ln\left(1 + COV^2_{\overline{c_u}}\right) + \alpha\beta\sqrt{ln\left(1 + COV^2_{\overline{c_u}}\right)}\right] \tag{24}$$

Here, the values on $COV^2_{\overline{c_u}}$ are given by the statistical analyses presented in section 4.2, β is given by standards and α is evaluated from reliability analyses, which are described further in Chapter 5.

Chapter 5 – Reliability-based design

In geotechnical engineering, soil properties are in general dealt with in a deterministic way. Mean values are often considered for design, and the effect of variation and fluctuation in these values is represented by a partial factor or a total factor of safety. A more rational approach for dealing with the variability and fluctuation of design properties is to use reliability-based design (RBD). There are several ways of carrying out a reliability analysis. In this study the Hasofer-Lind approach, also known as the first order reliability method (FORM), will be used. The first part of this chapter gives an example of an established deterministic design methodology for the serviceability limit state design of soil improved by lime-cement columns. The second part shows how the deterministic design methodology can be incorporated in a RBD methodology.

5.1 Serviceability limit state design

The current design methodology is described in TK Geo (2011), Larsson (2006) and SGF (2000). For simplicity, a simple settlement model that is easy to understand was used:

The total settlement (s) of an embankment founded on normally consolidated clay improved by end-bearing lime-cement columns can be described by:

$$s = \sum \frac{h_j \cdot q}{a \cdot E_{col} + (1-a) \cdot M_{clay}} \tag{25}$$

where h_j is the height of layer j, q is the additional strain, a is the area ratio for the lime-cement columns, E_{col} is the elastic modulus of the columns and M_{clay} is the oedometer modulus of the clay. The elastic modulus of the lime-cement column is normally not measured in-situ. It is instead assumed to be a function of the c_u evaluated and is assessed using (TK Geo 2011):

$$E_{col} = 13 \cdot {c_u}^{1.6} \tag{26}$$

Evaluating E_{col} from c_u introduces further transformation errors, which have to be considered in RBD.

The advantage of starting with a simple settlement model is that it makes it easier to focus on the reliability-based design methodology, rather than focusing on the complexity of the settlement model itself.

5.2 First order reliability methods ($FORM$)

Reliability analysis is an attempt to quantity how close a system is to failure (Baecher and Christian 2003). Failure in SLS can be defined as an unacceptable difference between expected and observed performance. To analyze the reliability of a geotechnical structure, a limit state function ($g(X)$) is defined at $g(X) = 0$. In SLS , $g(X)$ can be defined as:

$$g(X) = \delta_{max} - \delta(x_1, x_2, x_3 \dots) = 0 \tag{27}$$

where δ_{max} is the maximum settlement allowed and $\delta(X)$ is the settlement assessed from design properties $x_1, x_2, \dots, x_n$. $G(X) > 0$ indicates acceptable differences between expected and observed performance. By combining Equation 22, 25 and 27, the performance function can be re-written as:

$$g(E_{col}, M_{clay}) = \delta_{max} - \sum h_j \cdot \frac{q}{a \cdot (\mu_{E_{col}} - \alpha_{E_{col}} \cdot \beta \cdot \sigma_{E_{col}}) + (1-a) \cdot (\mu_{M_{clay}} - \alpha_{M_{clay}} \cdot \beta \cdot \sigma_{M_{clay}})} \tag{28}$$

where $\mu_{E_{col}}$ is the mean value of E_{col}, $\alpha_{E_{col}}$ is the evaluated sensitivity factor, $\sigma_{E_{col}}$ is the reduced standard deviation of E_{col}, $\mu_{M_{clay}}$ is the mean value of M_{clay}, $\alpha_{M_{clay}}$ is the evaluated sensitivity factor and $\sigma_{M_{clay}}$ is the reduced standard deviation of M_{clay}. Furthermore, q is considered to be deterministic. The reliability index is associated with the probability of failure and is determined by standards. The sensitivity parameter is given by an iterative process described by Rackwitz and Fiessler (1978) and Baecher and Christian (2003) and is defined as:

$$\alpha_{x_i} = \frac{(\frac{\partial g}{\partial x_i})}{\sqrt{\sum(\frac{\partial g}{\partial x_i})^2}} \tag{29}$$

where $(\frac{\partial g}{\partial x_i})$ is the partial derivate of $g(X)$ with respect to failure point x_i.

The derivation of Equation 28 shows the relative simplicity of combining a RBD methodology with an established deterministic design methodology.

Chapter 6 – Summary of appended papers

This licentiate thesis is based on three papers, which have been published in or submitted to international scientific journals. The following chapter is a summary of these papers.

6.1 Paper I

Strength variability in lime-cement columns based on CPT data

Mohammed Salim Al-Naqshabandy, Niclas Bergman and Stefan Larsson

Published in Ground Improvement 165(1), 2012

The aim of this paper is to describe the statistical parameters needed to quantify the spatial variability of soil improved by deep mixing. These parameters – the mean, the variance and the scale of fluctuation – are prerequisites for reliability-based design, which is a rational approach to incorporate uncertainties in a design. This study is based on 30 cone penetration tests in soil improved by deep mixing. The spatial variability with respect to the cone tip resistance is analyzed, and the scale of fluctuation is put into context with the variance reduction factor. The scale of fluctuation was estimated to 0.2-0.7 m and 0-3 m in the vertical and horizontal direction, respectively. A simple design consideration shows the potential influence of the variance reduction factor in determining the design value.

6.2 Paper II

Strength variability in lime-cement columns evaluated using CPT and KPS

Niclas Bergman, Mohammed Salim Al-Naqshabandy and Stefan Larsson

Submitted to Georisk *in November 2011*

This paper evaluates the strength variability in soil improved by deep mixing using two different test methods, the column penetration test and the cone penetration test. The study is based on 38 column penetration tests and 38 cone penetration tests, executed on two different test sites. Test data were quantified by means, variances and scale of fluctuations. The aim of this study is to examine the impact of each method in assessing the design value using reliability-based design. Uncertainties associated with design are discussed and a simple design consideration demonstrates the impact of different uncertainties in assessing the design value. The results from the analyses suggest that the relation between measured cone tip resistances from the cone penetration test and the column penetration test does not correspond to the cone factors proposed in previous studies and in the Swedish Design Guidelines. Finally, reliability-based design is recommended for both contractors and clients, since it promotes improvement in manufacturing methodologies and design models.

6.3 Paper III

Agreement between KPS and Jbt data evaluated from soil improved by lime-cement columns

Niclas Bergman and Stefan Larsson

Submitted to Ground Improvement *in April 2012*

In Sweden, the penetration test method commonly used for tests in lime-cement columns is the column penetration test. Because of the relatively large size of the test probe, it is recommended for depths no more than 8 m. At greater depths and in high-strength columns, the probe easily deviates from the column. To facilitate the verticality of the probe, a center hole can be bored in the column. This is usually done using the total-sounding test method. Consequently, two sets of test data are often produced for each column. The aim of this paper is to quantity the agreement between the two methods. If a good agreement is found, it should be possible to replace the column penetration test with the less expensive and less time-consuming total-sounding test. In this study, a good enough agreement between the methods was found. Thus, it is suggested that the total-sounding test be used as a complement to the column penetration test in evaluating the average strength properties of a group of medium- and high-strength lime-cement columns. However, in this study the tests were executed in medium- and high-strength columns. Accordingly, this study has not been able to quantify the agreement in the low-strength interval (undrained shear strength < 150 kPa).

Chapter 7 - Conclusions and future research

The following section is a summary of the major findings and conclusions of this study. In addition, future related research is suggested.

The spatial variability parameters, coefficient of variation (COV) and scale of fluctuation (θ) were evaluated on two different test sites, Lidatorp and E18. At Lidatorp COV with respect to CPT tip resistance ranged from 0.22 to 0.67, while the vertical and horizontal scale of fluctuation ranged from 0.2 to 0.7 m and 0 to 3 m, respectively. The coefficient of variation of CPT and KPS tip resistances at the E18 test site ranged from 0.18 to 0.59 and 0.19 to 0.47, respectively. The vertical scale of fluctuation evaluated from CPT and KPS measured 0.4 m and 0.6 m, respectively. The horizontal scale of fluctuation was evaluated at three depths for both CPT and KPS. At a depth of 2.5 m below ground, CPT data indicated a horizontal θ of 4 m. However, due to the scatter in the sample ACF, the evaluated horizontal θ is questionable. Furthermore, no indication of a horizontal θ could be found at any other depth or by using KPS. The wide range of COV indicates high variability in lime-cement columns. This variability is most likely due to the complex mixing process and to the inherent variability of the unimproved soil. It is recommended that spatial variability be considered in the RBD of soil improved by lime-cement columns. Furthermore, to fulfill the requirement of uncorrelated samples, tests should be separated with a distance greater than the scale of fluctuation.

Uncertainties in the evaluation of the design value are discussed and categorized according to their origin. As shown, the evaluation of the design value is strongly influenced by uncertainties due to transformation or model errors; consequently it is recommended that these uncertainties be considered in the RBD of soil improved with lime-cement columns. A major source of transformation errors is the empirical relations between tip resistances and column undrained shear strength. Therefore, it is important to calibrate the penetration test methods with a standardized method and, in so doing, reduce the transformation errors.

The relation between cone tip resistances measured from the cone penetration test and the column penetration test does not correspond to the cone factors proposed in previous studies and in the Swedish Design Guidelines.

In this study, a good enough agreement between Jbt and KPS was found, and it is suggested that Jbt be used as a complement to KPS in evaluating the average strength of a group of medium- and high-strength lime-cement columns. On sites where relatively good agreement between the methods can be shown, the number of KPS can be reduced. It is our belief that the use of Jbt can result in a more cost-effective testing methodology. However, due to the high variability in test data, Jbt should not be used to evaluate single point values. The correlation and agreement analyses were performed on test data from medium- and high-strength columns (undrained shear strength > 150 kPa). Consequently, the study has not been able to quantify the correlation and agreement in the low-strength interval (undrained shear strength < 150 kPa).

Future research that will be conducted within the scope of this doctoral project:

- A reliably-based design model for lime-cement columns in serviceability limit state will be developed and presented.
- To facilitate the implementation of a RBD methodology, benchmark calculations with partial factors associated with DM design property variability will be developed and presented.

Future research considered to be beyond the scope of this doctoral project:

- Due to the strong influence of transformation and model errors, it is important to minimize the magnitude of these errors. Accordingly, it is recommended that the column penetration test be calibrated using a standardized method and, in so doing, reduce the transformation errors.
- This study has not been able to quantify the correlation and agreement between Jbt and KPS in low-strength columns (undrained shear strength < 150 kPa). Accordingly, further research on the correlation and agreement between the two methods is needed.
- Because of the significant influence of Jbt bar friction on the magnitude of the total penetration force, there is a need for a standardized method for Jbt bar friction assessment.
- Spatial variability has a significant influence in determining the design value. This variability could be reduced if a better understanding of the evaluation of spatial variability under different conditions (such as amount of binder, rotational speed of the mixing tool, penetration test method used) is achieved.

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